

Evaluation of Energy Based Pushover Analysis Procedure for Reinforced Concrete Frame Structures

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Abstract:- The recent development in the computer application has helped the structural engineering field significantly. The amount of time and efforts required for the analysis decreased drastically with the development of civil engineering software's. The Non-Linear Time History Analysis (NL-THA) of seismic evaluation of a structure is precise, but tedious and requires a lot of data and human skill to perform. Research has been put in to develop Non-Linear Static Procedures which can yield results close enough to the NL-THA procedure. The present study is based on Energy based pushover analysis, with considering the effects of higher modes on the response of the structure. The RCC moment resisting frames considered, were loaded and designed according to IS-1893-2002 (Part-1). The structures were analyzed for monitored roof-displacement of 4% of the total height. Existing nonlinear static (pushover) methods of analysis establish the capacity curve of a structure with respect to the roof displacement. Disproportionate increase in the roof displacement, and even outright reversals in the case of higher modes pushover analyses, can distort the capacity curve of the equivalent SDOF system. Rather than viewing pushover analyses from the perspective of roof displacement, this paper considers the energy absorbed (or the work done) in the pushover analysis. Simple relations are developed in energy-based displacement that is equivalent to the spectral displacement obtained by conventional pushover analysis methods within the linear elastic domain. Extensions to the nonlinear domain allow pushover curves to be established that resemble traditional first mode pushover curves and which correct anomalies observed in some higher mode pushover curves.

I. INTRODUCTION

The purpose of the pushover analysis is to evaluate the expected performance of a structural system by estimating its strength and deformation demands in design earthquakes by means of a static inelastic analysis, and comparing these demands to available capacities at the performance levels of interest. The evaluation is based on an assessment of performance parameters, like global drift, inter-storey drift, inelastic element deformations (either absolute or normalized with respect to a yield value), deformations between elements and connection forces. The inelastic static pushover analysis can be viewed as a method for predicting seismic force and deformation demands, which accounts in an approximate manner for the redistribution of internal forces occurring when the structure is subjected to inertia forces that no longer can be resisted within the elastic range of structural behavior. The pushover is expected to provide information on many response characteristics that cannot be obtained from an elastic static analysis. The use of pushover analysis methods for

characterizing the predominant mode of response of structures responding nonlinearly to earthquake ground motions has become well established. Approximate and exact first mode analysis procedures have been accepted in documents such as ATC-40^[1] and FEMA-273^[2]. In pushover analysis procedures, the behaviour of the structure is characterized by a capacity curve. In nearly all cases, the capacity curve is a plot of the base shear force versus the displacement of the roof. The seismic demands are determined throughout the structure based on the peak roof displacement estimated in each of the modal pushover analyses. When nonlinear behaviour develops in the pushover analysis, the displacements of the floors and roof will increase disproportionately with increasing load, in general. The arbitrary choice to plot the base shear as a function of the roof displacement introduces an arbitrariness to the inelastic portion of the capacity curve. For systems with sharply defined yield points, disproportionate increases in displacements over the height of the building, primarily affects the post-yield stiffness of the capacity curve. Because small deviations in the post-yield stiffness of the capacity curve of the "equivalent" single-degree-of-freedom (SDOF) system typically have only minor effects on the dynamic response statistics, any departures from theoretically ideal values can be difficult to discern in computational studies. Where yielding is more gradual, disproportionate increases in the roof displacement may, in addition, affect the effective yield strength that is determined for the structure, when methods such as those described in ATC-40^[1] are used. Roof displacements may increase at a decreasing rate or may even reverse (Fig.1), leading to capacity curves that display unusual behaviour, a literal interpretation of the capacity curves obtained in these cases would indicate that the structure does not always absorb energy in a pushover analysis, but instead, may be a source of energy for some inelastic regimes (Hernandez-Montes, E., et.al.,^[3]). Such an interpretation implies a violation of the first law of thermodynamics, and points out the degree to which the use of the roof displacement can be misleading. There is no doubt that external work is consumed by the deformations of plastic hinges (and any changes in recoverable strain energy) that take place in a monotonic pushover analysis. The notion that the structure may be a source of energy is a consequence of the arbitrary choice to use the roof displacement as the index (abscissa) of the capacity curve.

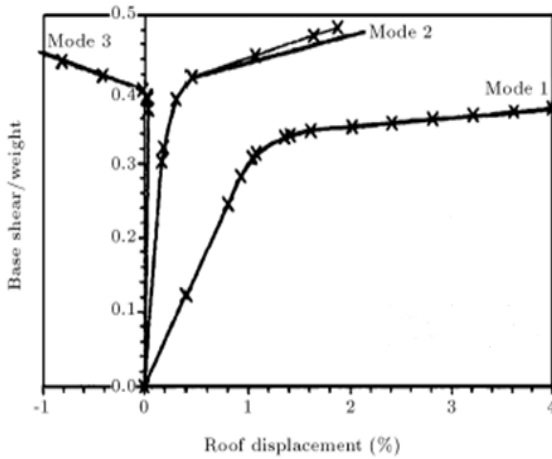


Fig.1 Reversal of Pushover Curve in Mode 3

II. ENERGY BASED APPROACH FOR PUSHOVER ANALYSIS

At its core, the capacity curve of a structure represents the development of resistance to lateral forces as a function of increasing lateral displacements. The capacity curve has great value in characterizing the degree of nonlinearity that may develop in a first or predominant "mode", recognizing, of course, that the onset of nonlinearity causes changes in modal properties and invalidates modal superposition. Because floor displacements over the height of the building generally increase disproportionately as the response becomes increasingly nonlinear, one cannot rigorously justify the use of the displacement at any one location for the abscissa of the capacity curve, since the apparent post-yield stiffness of the capacity curve depends on the location selected. As shown outright reversals in the capacity curve may result in some cases. Rather than relying on the roof displacement the use of energy absorbed by the structure in each modal pushover analysis to determine the corresponding capacity curve of the equivalent SDOF system, recognizing that the behaviour of the MDOF system and its analogous SDOF system can be appreciated from both conventional and energy-based perspectives. The energy-based formulation developed below avoids the arbitrary selection of a single floor (or roof) location as the parameter for representing the capacity curve, and may be used with single or multimode analysis procedures.

The equation of motion is often expressed as the dynamic equilibrium of force quantities but can equivalently be expressed in terms of energy quantities.

$$m\ddot{u} + c\dot{u} + ku = -Im\ddot{u}_g(t) = p_{eff}(t) = -S\ddot{u}_g(t) \quad \dots(1)$$

The "absolute" energy form of Eq. [1] expressed in terms of the energy developed from the time that the excitation starts, can be obtained by integrating Eq. [1] with respect to displacement:

$$\frac{1}{2}\dot{u}_t^T m \dot{u}_t + \int \dot{u}_t c du + \int f_s^T du = \int (\sum_{i=1}^N m_i \ddot{u}_{ti}) du_g \quad \dots(2)$$

where m_i is the lumped mass associated with the i^{th} story and it is the absolute (or total) acceleration at the i^{th} story, and f_s is the restoring force.

In both the "absolute" and "relative" energy formulations of the equation of motion, the absorbed energy, E_a is

$$E_a = \int f_s^T du \quad \dots(3)$$

The absorbed energy is composed of the recoverable elastic strain energy and non-recoverable energy associated with energy dissipated by the hysteretic response of the structural components. The static force associated with the n th mode is $f_n(t)$. The restoring force is assumed to be equal to sum of the modal components $f_n(t)$. Following this assumption, the restoring force f , can be represented in terms of its modal components:

$$f_s(t) = \sum_n f_n(t) = \sum_n \omega_n^2 m \varphi_n \Gamma_n D_n(t) \quad \dots(4)$$

Due to the orthogonality of modes with respect to k , the force f_n does work only for displacements in the n th mode. The work done by this force on the other modal displacements is zero. In the elastic domain, the absorbed energy associated with the static force f_n going through an elastic displacement from 0 to u_n may be computed as:

$$E_n = \frac{1}{2} f_n^T \cdot u_n = \frac{1}{2} \omega_n^2 \varphi_n^T m \varphi_n \Gamma_n^2 D_n^2(t) = \frac{1}{2} \omega_n^2 \Gamma_n^2 M_n D_n^2(t) \quad \dots(5)$$

The corresponding base shear associated with the n th mode pushover is:

$$V_{bn} = f_n^T \cdot 1 = \omega_n^2 \Gamma_n \varphi_n^T m 1 D_n(t) = \omega_n^2 \Gamma_n^2 M_n D_n(t) \quad \dots(6)$$

Substituting Eq. (6) into Eq. (5), we obtain

$$E_n = \frac{1}{2} V_{bn} \cdot D_n(t) \quad \dots(7)$$

for the response in the elastic domain.

Equation (7) can be interpreted graphically as the area beneath the curve in a plot of $V_{b,n}$ with respect to D_n in the elastic domain. Therefore, we define the energy-based displacement, $D_{e,n}$ to be equal to $2E_n/V_{b,n}$ in order to assure that $D_{e,n} = D_n$ in the elastic domain. More generally, for both the elastic and inelastic response, the work done by $V_{b,n}$ in a differential displacement $dD_{e,n}$ is dE_n :

$$dE_n = V_{bn} \cdot dD_{e,n} \quad \dots(8)$$

which is necessarily equal to the work done by the static force f , in a differential displacement of the structure in this mode. Using an incremental formulation, the terms ΔE_n and V_{bn} can be computed for each step in the pushover analysis. Then, the corresponding increment in the energy-based displacement, $\Delta D_{e,n}$ may be calculated as:

$$\Delta D_{e,n} = \frac{\Delta E_n}{V_{bn}} \quad \dots(9)$$

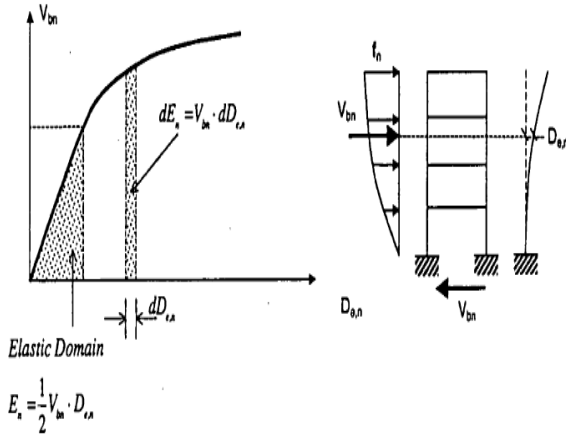


Fig. 2 Calculation of Energy

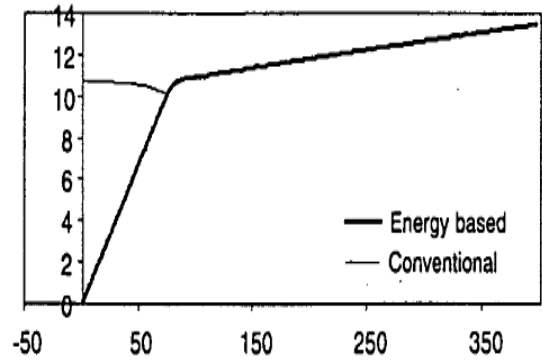


Fig.3 Comparison of Pushover Curves

III. CASE STUDY

Four R.C Symmetrical Space Structures with varying no. of stories are modeled in ETABS. They are initially analysed and designed for D.L, L.L and Seismic Loads and their combinations according to I.S. Codes. The

various parameters considered in the design are presented in the Table-1 & the plan is shown in Fig.4. Fig. 5 shows the typical model of the 4 storey frame in ETABS.

Table I Assumed Preliminary data required for the Analysis of the frame

Sl.no	Variable	Data
1	Type of structure	Moment Resisting Frame
2	Number of Stories	4, 8, 12 & 20 Storey
3	Floor height	3.2m
4	Live Load	3.0 kN/m ²
5	Floor Finish Load	1.0 kN/m ²
6	Materials	Concrete : M25 and Steel : Fe415
7	Size of Columns	300x450 mm
8	Size of Beams	300x450 mm
9	Depth of slab	120mm thick
10	Specific weight of RCC	25 kN/m ³
11	Zone	V
12	Importance Factor	1
13	Response Reduction Factor	5
14	Type of soil	Medium

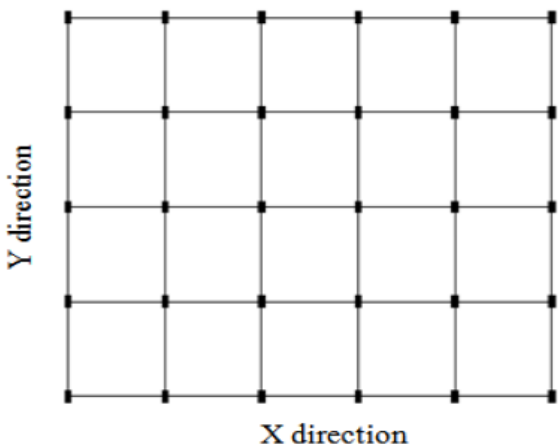


Fig.4 Plan of the Building

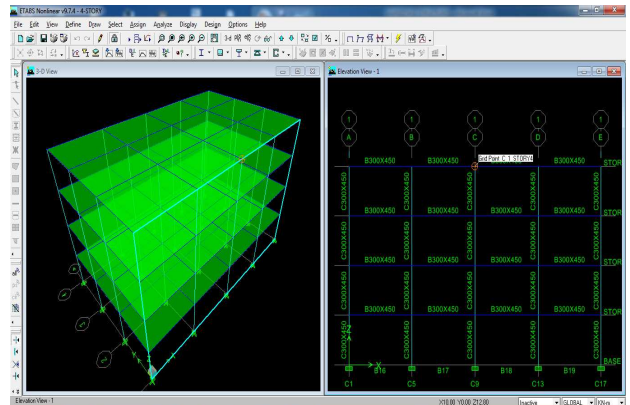


Fig. 5 Building Model in ETABS

A. Pushover Analysis

Pushover analyses were performed on the frames using ETABS^[8]. In pushover analyses, invariant lateral load pattern proportional to Mode Shapes of the buildings were applied. The pushover curves obtained for different mode shapes are noted directly from the ETABS. It is observed that in some cases of pushover analysis disproportionate increase in the roof displacement, and even outright reversals in the case of higher mode pushover analyses, which is distorting the capacity curve of the frame.

B. Energy Based Pushover Analysis

From the results obtained in the pushover analysis and using the equations derived in the section II, energy based displacements are calculated. Table II shows the Base shear and displacement values of the first mode pushover analysis of the four storey frame. Using these values energy based displacements are calculated which are shown in Table III. Plotting these Displacements Vs Base Shear, a new pushover curves called as Energy based Pushover curves are obtained. It

is seen that the pushover curves which were getting reversed in the conventional pushover analysis are maintaining their profile in the same direction. The following section shows the comparison of pushover curves between conventional pushover analysis and energy based pushover analysis.

In these plots the values expressed are obtained from the equations proposed by Hernandez Montes^[3] in the EBPOA, while the results for MPA have been obtained directly from the ETABS^[8]. It is seen that the energy based curve for the first two modes response nearly coincides with the conventional pushover capacity curves that is determined using the ATC-40^[1] procedures. The use of the roof displacement in the conventional MPA approach leads to an apparent stiffening in the post – yield response, while the energy-based approach shows monotonic softening with increasing displacement. Figs 6e, 7e, 8e & 9e shows the curves generated for the higher modes of analysis; the energy based method rectifies the mode reversal apparent with the conventional roof-displacement approach (Figs 6f, 7f, 8f & 9f).

Table II Base Shear and Roof Displacement for the First Mode Pushover Analysis

Mode-1		
Step No	Displacement (m)	Base Force (kN)
0	0	0
1	0.0104	513.0443
2	0.0219	866.1347
3	0.0256	930.6448
4	0.0426	1055.4941
5	0.0463	1066.8301
6	0.0564	1082.1888
7	0.1134	1131.4758
8	0.1266	1137.1489
9	0.1664	1139.8301

Table 3 Calculation of Energy Based Displacement

STEP	0	1	2	3	4	5	6	7	8	9
$\Sigma \Delta W$	2.10	6.47	2.76	15.69	4.85	14.54	62.10	12.47	38.52	0.27
ΔU_{en}	0.0082	0.0093	0.003	0.015	0.004	0.013	0.056	0.01	0.03	0.0002
U_{en}	0	0.0093	0.012	0.028	0.032	0.046	0.102	0.11	0.14	0.14
B.S	0	513.04	866.13	930.64	1055.49	1066.83	1082.18	1131.47	1137.14	1139.83

IV. RESULTS

The following curves show the comparison of pushover curves for different mode shapes of the structure. Fig. 6a, 6c and 6e shows the pushover curves obtained from

conventional pushover analysis for 4 storey building where as Figs. 6b, 6d and 6f represents its equivalent through Energy based pushover analysis. Similar comparisons are shown for 8-storey, 12-Storey and 20 Storey building.

4-STOREY BUILDING:

i) Mode#1(X-Direction):

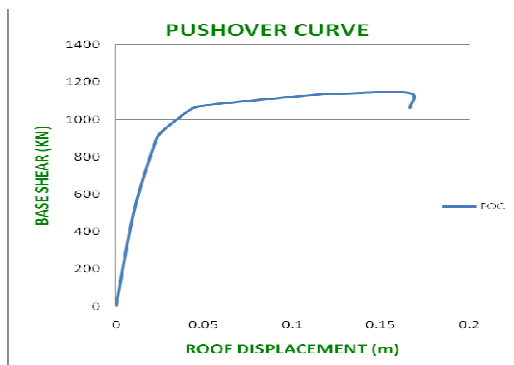


Fig.6a

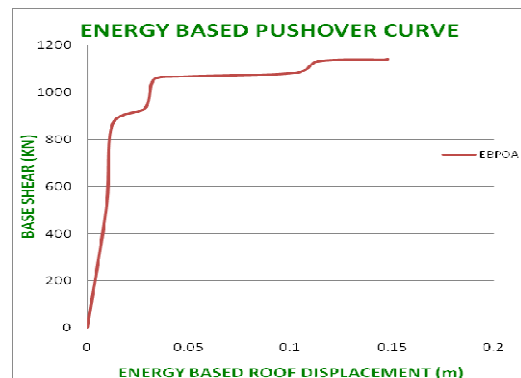


Fig.6b

ii) Mode#2(Y-Direction):

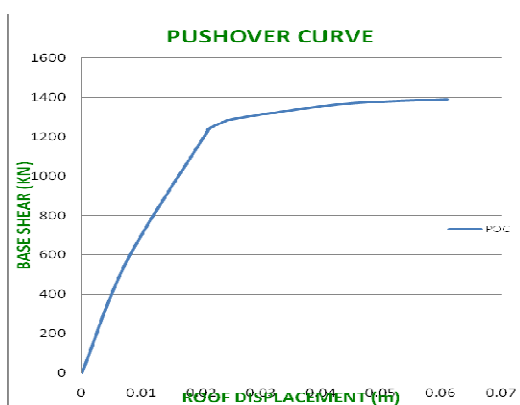


Fig.6c

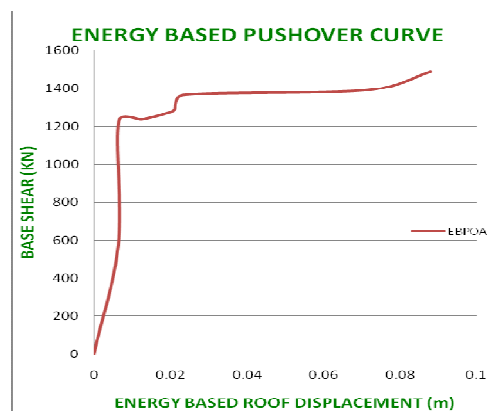


Fig.6d

iii) Mode#21 (X-Direction):

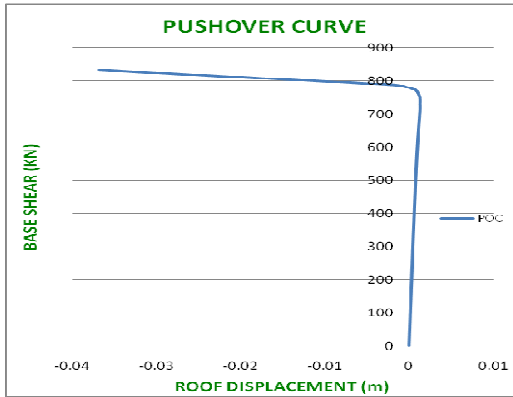


Fig.6e

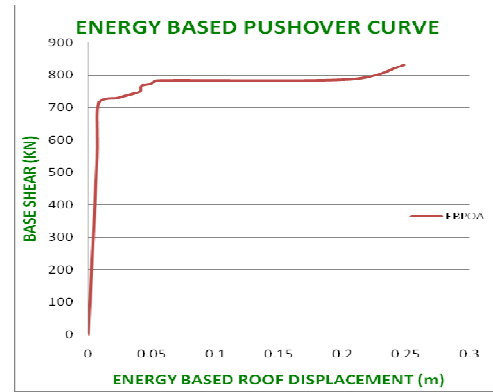


Fig.6f

8-STOREY BUILDING:

i) Mode#1(X-Direction):

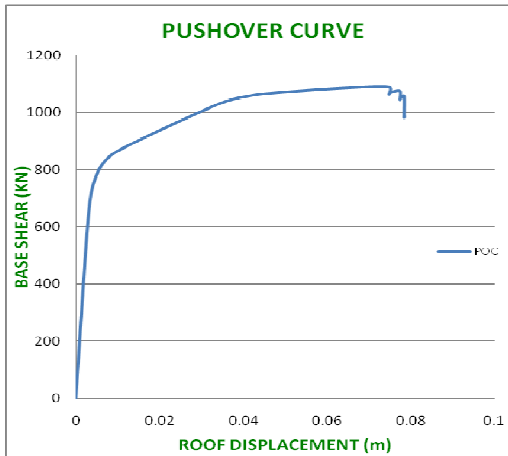


Fig.7a

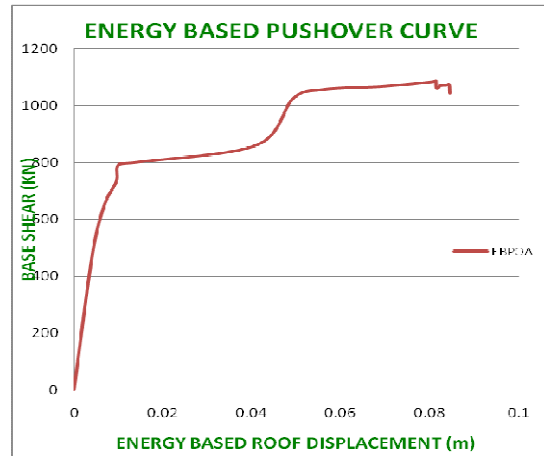


Fig.7b

ii) Mode#2(Y-Direction):

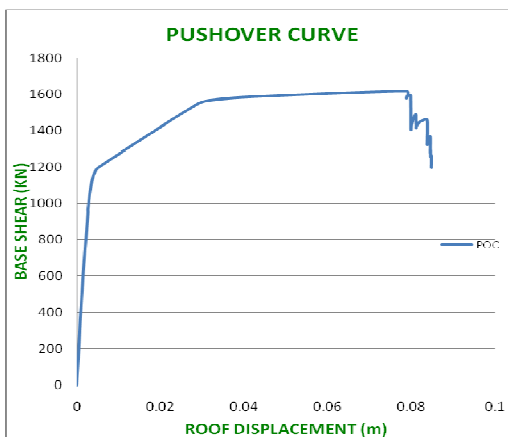


Fig.7c

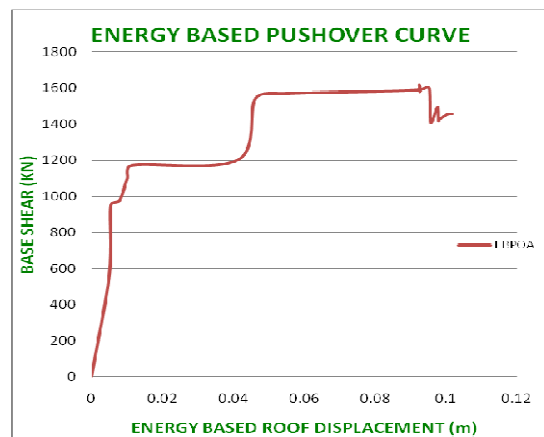


Fig.7d

iii) **Mode#15(Y-Direction):**

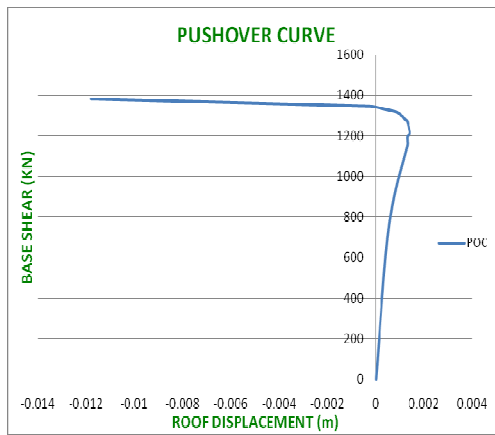


Fig.7e

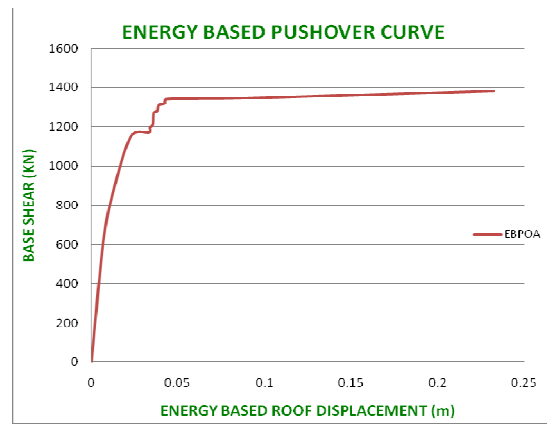


Fig.7f

12-STOREY BUILDING:

i) **Mode#1(X-Direction):**

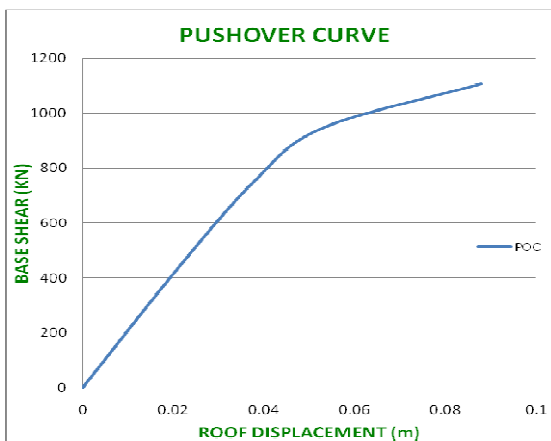


Fig.8a

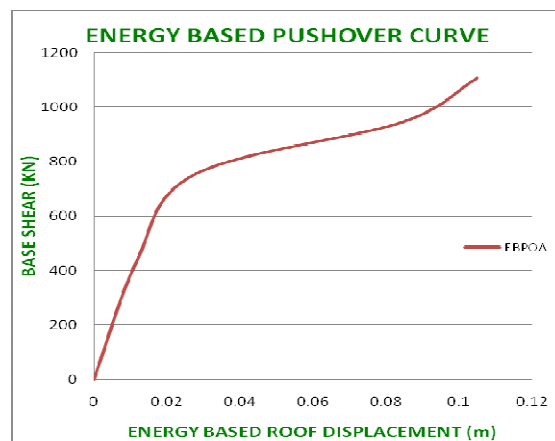


Fig.8b

ii) **Mode#2(Y-Direction):**

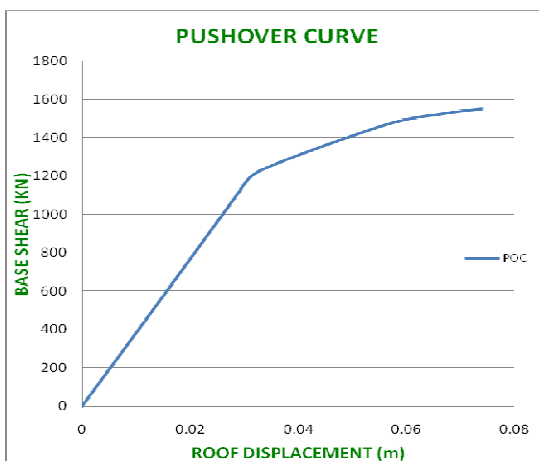


Fig.8c

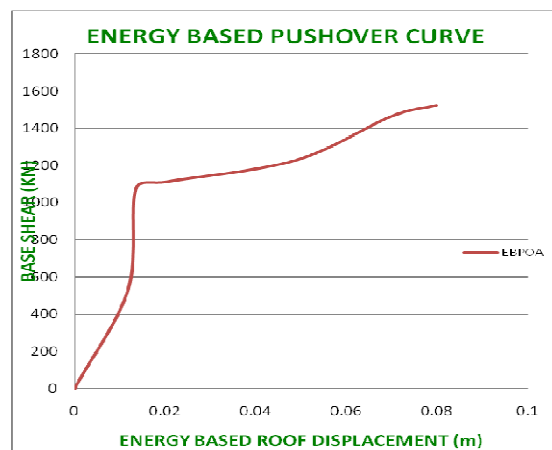


Fig.8d

iii) Mode#12(Y-Direction):

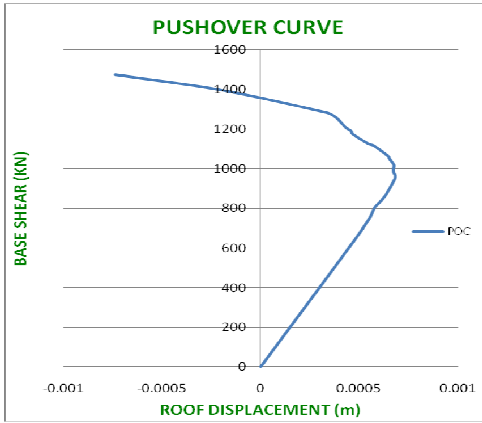


Fig.8e

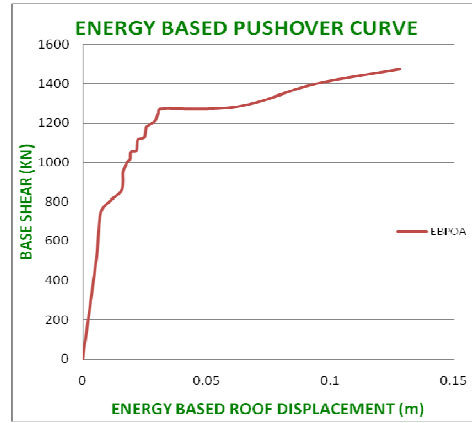


Fig.8f

20-STOREY BUILDING:

i) Mode#1(X-Direction):

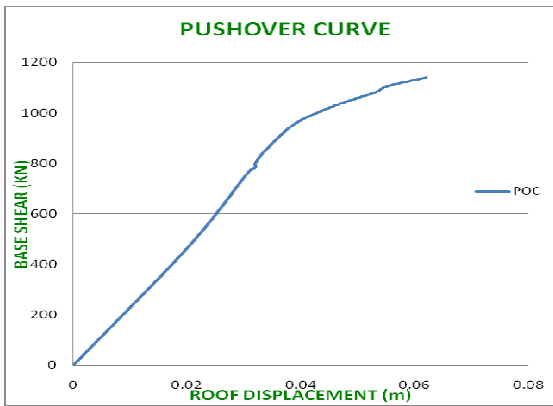


Fig.9a

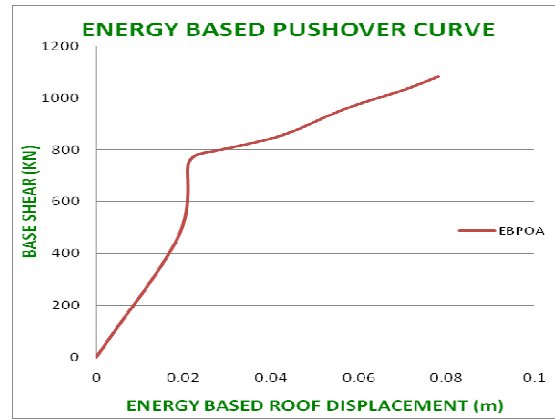


Fig.9b

ii) Mode#2(Y-Direction):

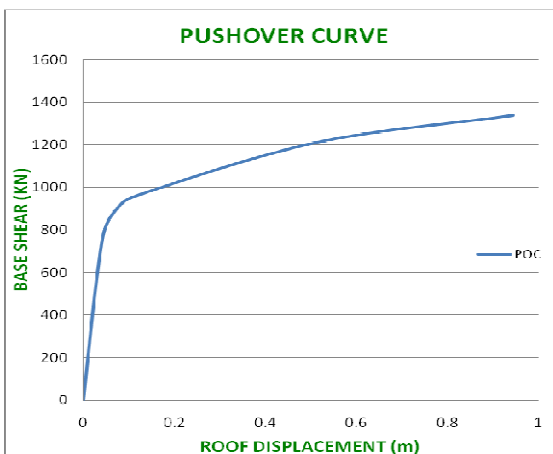


Fig.9c

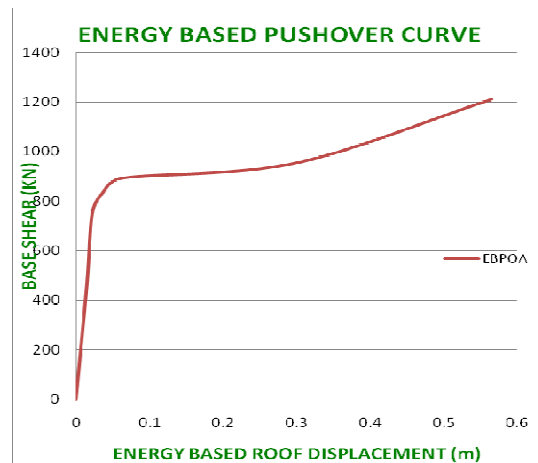
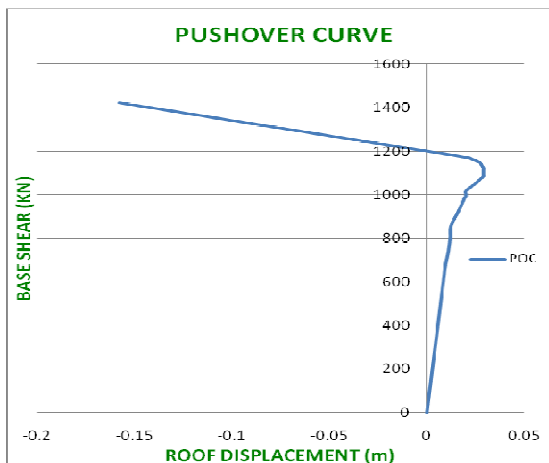
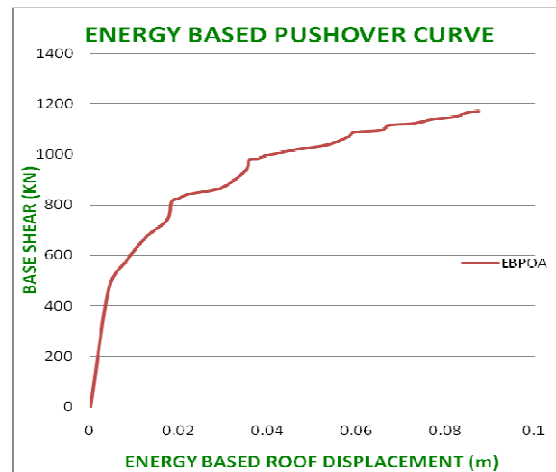


Fig.9d

iii) Mode#9(Y-Direction):

Fig.9e

Fig.9f
V. CONCLUSIONS

- Energy based pushover curves shows a gradual softening of the structure rather than a quick failure which is observed in conventional pushover analysis
- EBPOA overcomes the major drawback of reversal of capacity curves in the conventional analysis.
- With the total work done in the structure involved, this method provides better results for the structural evaluation under seismic loading and for consideration of higher mode effects.
- As the total energy absorbed is considered, the method doesn't overestimate the response of the structure like MPA.
- Reversal of Pushover curves tends to occur quickly as the number of stories increases.

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